CHAPTER 6

REFERENCE MATERIAL

Background material

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The Coevolving Organization described how to bring a business to an optimal degree of decoupling on the order–chaos boundary, but stopping just short of the boundary to imbue some measure of stability. However, HOT has – as described earlier in the present text – allowed this optimal point to be pushed further towards the chaotic regime provided we know the likelihood of initiating chaotic events: this enables us to limit (buffer) their effect on the rest of the business. But it is also possible under some circumstances to operate *within* the chaotic regime and to control what happens.

A chaotic system typically contains one or more fuzzy (imprecisely defined) areas – 'strange attractors' – to which movement gravitates. Irrespective of starting point (i.e. whatever the initial conditions), subsequent paths are inexorably drawn in to an attractor, although they may well cycle around the attractor following some convoluted route for ever. Some paths may be simple periodic ones like the moon going around the earth, although less stable; others may be extremely complex where the system loops through a sequence of different orbits before repeating the whole series; these are called 'high-period' orbits. Methods for controlling chaos are based on one or both of a couple of observations by Grebogi et al (reference **4**): that most chaotic attractors contain an infinite number of unstable periodic orbits, and that each such orbit contains a series of saddle points. The relevant feature of a saddle point is that on one axis – forwards and backwards on a real saddle – movement is stable (the rider sinks towards the centre of the saddle), while on the other axis – side to side on a real saddle – movement is unstable (the rider falls off!). Ott et al (reference **9**) made use of the existence of saddles and their stable/unstable behaviour to optimize the behaviour of the system by firstly pinpointing where a suboptimal unstable trajectory (associated with some particular start point) approached a more optimal path. Then, in the area where the paths ran adjacent to (or crossed) each other, they proposed nudging the system to shift from the undesirable path to the better path. This made the dynamics of the system work in their favour because a *small* perturbation in the right direction on to the stable part of a saddle then allows the natural stabilizing behaviour of the front-back axis of the saddle to take over and complete the job. The process, subsequently known as the OGY method, does not correct continually but instead applies an intermittent correction to some system-wide control variable (i.e. adjustment knob) which controls the path of the trajectory as it cycles around its attractor. A correction is made once per cycle, and the size of the correction is calculated such that the *next* time the trajectory comes around in its cycle, it hits the stable area of the saddle and thus of its own volition is dragged into the desired optimal orbit. More precisely, since adjusting the control variable adjusts the entire system and not just the current trajectory, the effect moves *all* the possible orbits a small amount (by analogy: instead of stepping sideways on a carpet to stand on some better spot, one moves the whole carpet instead – which affects everyone standing on the carpet…). When the path of the system is shifted to the optimal orbit in this way, subsequent unpredictable external buffeting may make the system change course off its (new) optimal orbit. In this case, nudges can be reapplied to keep the system on course. OGY has a couple of disadvantages, however. The first is that because corrections are only made intermittently rather than continually, these corrections are inevitably jerky, albeit small, and between one correction and the next the trajectory might have veered off course due to noise. The second is that calculating the correction needs to be done very quickly in order to apply it, and this may not be possible if the system itself changes quickly (has high frequency oscillations, for example).

An alternative (the delayed feedback control or 'DFC' technique) based on OGY and due to Pyragas (references **11** and **12**) gets around both these problems. Pyragas proposed applying a correction based on the difference between the value of some system variable now and its previous value. The delay between the measurement of the 'now' value and the immediately previous value is set to be the orbital period of the desired path. The correction thus pushes the system to have the same orbital period as the desired path. Two major differences between the Pyragas approach method and OGY is that the Pyragas correction adjusts only the current trajectory and not the whole system, and that corrections are applied continuously whereas OGY applies corrections not just once per cycle but also 'in advance' (OGY has to predict where the current trajectory will be next time around and correct accordingly). A cross between OGY and DFC also exists (see Bielawski et al reference **2**): it uses OGY's once-per-cycle adjustment but DFC's delays to calculate the adjustment.

There are several versions of the OGY and Pyragas methods, but most apply a corrective adjustment whose size is roughly proportional the gap between the current path and the desired path provided the gap is not too large. Since large adjustments must be avoided since they may perturb the already chaotic system into new and uncharted territory, either the adjustment is bounded (is within a 'window' with upper and lower limits) or a small adjustment is repeated regularly until it has the required effect (see OPF below). Some methods (see e.g. Arecchi and Boccaletti reference **1**) also measure the rate at which the trajectory is diverging from the desired UPO (i.e. the trend as well as the current difference) and increase the adjustment further if the trend is particularly adverse or reduce it if the system appears to be responding well.

Socolar et al (reference **16**) significantly improved on the basic Pyragas method using a correction which was based not just on the difference between the current and immediately preceding value but also on a weighted sum of differences going back in time ('last minus last-but-one; last-but-one minus last-but-two… etc). Socolar's improvement is known variously as the extended DFC ('EDFC') or the extended time-delay auto-synchronization ('ETDAS') technique.

One problem with all of these techniques – but especially OGY – is that the initial suboptimal path needs to approach sufficiently near to the optimal one for a small adjustment to be effective, and so refinements have been developed which allow a succession of jumps from suboptimal path, via a better path, then via an even better path to the optimal path, each triggered by a small adjustment. The resulting path is thus built up from sections of larger paths and looks like (and is called) a 'bush'. Shinbrot (references **13** and **14**) first described this 'targeting' process in detail, and it is of particular value when the chaotic system has more than one attractor and we want to persuade paths from the majority of starting points to go to just one of the attractors.

A second problem is identification of the desired periodic orbit. This is usually done by analysing time series data with methods proposed by Lathrop and Kostelich (reference **8)** and by So et al (reference **15**). The idea behind both is that, since we may well not know what the attractor looks like, we firstly reconstruct it using a series of measurements of some chosen system variable (a process called 'delay co-ordinate embedding'[1] first described in this context by Packard et al – see reference **10**) and then identify UPOs, their orbital periods and saddle points either graphically or by further analysing the data.

A third difficulty with OGY in particular arises when the orbits are 'high period' (multi-loop) rather than simple ones, since a correction calculated at the start of each sequence of loops may be hopelessly inaccurate by the time all the loops have been traversed and the sequence is ready to be repeated. Hunt (reference **7**) proposed measuring the error, calculating an adjustment proportional to the error (except when the error was too large, in which case no attempt at adjustment was made), and making the adjustment intermittently[2] – hence the technique's name of 'occasional proportional feedback' or OPF. Because the adjustment can be relatively large (larger than, for example, used by OGY), OPF can create periodic orbits where none previously existed.

These methods for the control of chaotic systems have been applied in practice to laboratory and industrial systems and to unmanned spaceflight. Boccaletti et al (reference **3**) is recommended as a comprehensive (95 page) up-to-date review of the various methods and their implementation. Use of chaos control within a business organization would be feasible in principle, but it presupposes that the organization is already chaotic and remains chaotic – these methods do *not* make a chaotic system unchaotic, merely better performing, less unstable and more predictable. Simple models of business competition do, however, exist (see for example Holyst references **5** and **6**).

[1] this remarkable and counter-intuitive result – that an attractor can be reconstructed using just a series of measurements of one variable – is a consequence of the Takens-Whitney embedding theorem, named after Dutch mathematician Floris Takens and US mathematician Hassler Whitney. The attractor thus reconstructed may not *look* exactly like the original but they can be 'morphed' from one to the other and both have the same dynamic properties

[2] Hunt's system was an electronic circuit rather than a computer model. The adjustment was continuous but for a short time, like pushing then briefly holding in a bell-push.

CHAPTER 7

QUESTIONS AND ANSWERS

Q: I think I understand the 'controlled percolation' forest fire formulation of HOT, but cannot see the connection between this and the probability – loss – resource (PLR) version. Does PLR occur in real-life?

A: The Duke of Wellington[3] was outnumbered when defending against the French at Torres Vedras (near Lisbon) during the Iberian Peninsular War. He had two conflicting constraints: *winning* while *minimizing casualties* (loss) and he was, with some restrictions, able to place his troops such that the probability of casualties *overall* was minimized. Some soldiers would be in advanced positions most likely to be attacked but Wellington ensured that these were in small groups heavily protected by gun emplacements, palisades and earthworks that were built at considerable cost by several thousand Portuguese labourers. At the other extreme, he spent little on protecting his reserves that were further from the firing line. Given this strategy:

- a 'normal' HOT formulation would be: win whilst minimising the cost of casualties *plus* the cost of flank protection (more small groups = more flanks to protect). The difficulty is that turning either casualties into money or the cost of flank protection into equivalent 'avoided casualties' is subjective.

- the PLR HOT formulation would be: win whilst minimising the cost of casualties subject to a *limit* on the cost of flank protection. His tactical problem was this: with a fixed-sized war chest for spending on defences, where should he spend the money on building these defences such that his overall casualties were minimized, *given his assessment on the likely casualties in each area*. In this formulation, there is no need to put a price on casualties.

The 'normal' formulation is thus:

- optimize yield where the yield (value) is offset by the cost of flank protection which insulates one area from another

  whereas the PLR formulation is:

- optimize yield subject to a limit on the total cost of flank protection

The first assumes no overt limit on the cost of flank protection, but assigns a cost such that the minimization process itself puts a brake on the amount of flank protection used. It makes a compromise between the value of the yield and the

[3] 1769-1852; the UK's best field commander since the (1st) Duke of Marlborough. Although at the time not yet a Duke, he was on fast track promotion during the war as progressively Sir Arthur Wellesley; Baron Douro; then Viscount, Earl and lastly Marquis of Wellington.

cost of protection. The second formulation does not assign any cost per unit length of flank protection, but limits the total amount that can be employed. From the above example, Wellington had a fixed army; his latitude was how to deploy them in groups geographically. More small groups limit the *overall* impact of a sudden and successful assault on his troops: some small regiments may be totally wiped out but, since he had deployed his troops such that the ones most at risk were protected by the best defences, Wellington had done his best[4].

Q: In *The Coevolving Organization*, you described the order–chaos boundary as the farthest point it was sensible to push the decentralization of decisions without all hell being let loose. You have now shown how HOT would allow me to reduce the coupling between parts of my organization even further (to the 'chaos' side of the order – chaos boundary) provided I can identify the risks of adverse behaviours in advance. The result will. I hope, be even better responsiveness while limiting the effect of any adverse decisions on the rest of the business. Remember Barings Bank (RIP)? If Barings had run its Singapore branch with a separate individual as head of settlement operations to control the exposure of Nick Leeson's trading, the branch would have remained under control and solvent. If, however, it were run with Leeson in both roles (as happened) but as an entirely separate company at arms length, the Singapore branch would have gone catastrophically bust but Barings – legally buffered from the exposure – would have survived (financially; but its good name would have been forever blackened). I accept that the impact of *unanticipated* risks such a Leeson's rogue trading is likely to be greater than if my organization stopped decentralizing when it hit the order – chaos boundary, and that I have consciously traded off greater fragility to the unexpected in order to give me greater effectiveness as a result of further decentralization. But responsiveness is everything in my business, and I would now like to decentralize decisions even further. Is there any way a business which deliberately operates in the chaotic regime can be controlled?

A: Possibly – it depends on the nature of the chaotic behaviour. Simple examples are difficult to come by, but you can get some idea of what is involved from observing the behaviour of things designed to be unstable. For example, fighter aircraft such as the Eurofighter Typhoon or the F-22 are engineered from the outset be unstable. This gives them great responsiveness and manoeuvrability but they also need complex control systems to enable any mortal to fly them. Formula 1 racing cars are not allowed this degree of 'fly by wire' electronic assistance, but they are still far less stable than the typical family saloon; they are exceptionally responsive and can change direction in an instant, but are difficult to drive. To construct our example of the control of (near-)chaotic behaviour, we need to couple a modern marginally-stable F1 racing car with one of the very few F1 drivers[5] who, during practice, could repeatedly drive same optimum path around the circuit. Assume that, unlike a current F1 car, the car can also be driven automatically – driverless – with the car's electronics remembering (and being

[4] he won…his defence and logistics were so good that the French under Marshal Massena, with very tenuous supply lines, themselves starved and retreated

[5] Argentina's Juan Manuel Fangio (1911 – 1995) and Scottish driver Jim Clark (1936 – 1968) were perhaps the greatest exponents of this 'driving on rails'

able to reproduce – a bit like a pianola) the exact sequence of accelerator, brake, gear-change and steering-wheel movements used by the driver in covering a perfect lap. Now let the car loose from the starting grid to cover a lap. As it progresses around the circuit, it will follow the built-in programme: accelerate immediately, brake after seventeen seconds, turn the steering wheel a quarter turn after nineteen seconds, and so on. When the car completes a circuit and crosses the start line again, it will inevitably not be in quite the same position on the track as when it started off: it may, for example, be a metre to the right-hand side of the optimum path. Small gusts of wind, tyre wear and a thousand-and-one other things will deflect it slightly from its optimal course.

To remedy this, we could do one of two things:

- either install an ultra-accurate GPS-like system to record exactly where on the track each acceleration, braking, gear-change or steering wheel movement should happen, and then, during the driverless laps, make *continual* minor corrections to speed and position so that the car follows the optimum path as exactly as possible

- or correct the speed and position *just once* – each time it crosses the start line

In either case, because the car is light and only marginally stable, changes of direction or speed need a mere touch of the steering wheel or accelerator.

So much for the example, but what about control of *chaotic* systems? If the movements in a chaotic system have some form of repetition, i.e. they are periodic (like the F1 car lapping the track), it is likely also that the instability can be made to work for us: a nudge in the right direction at the right time and the system itself will take over and automatically guide the movement to where we would like it to go without further effort on our part. A periodic chaotic system can thus be controlled in two ways: a *nudge to the controls* once each time around (the OGY method named after US physicists Ed Ott and Jim Yorke[6] and Brazilian physicist Celso Grebogi) or with a continuous correction – the Pyragas method named after Lithuanian physicist Kęstutis Pyragas.

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[6] In passing, it appears that Yorke (with colleague TY Li) may have been the first to use the word ‘chaos’ in its mathematical sense

BIBLIOGRAPHY

Books

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Michael Berthold, M. and Hand, D. J. *Intelligent Data Analysis* Springer (2003)
(contains inter alia an excellent introduction to delay co-ordinate embedding)

Schuster, G.S (ed.) "Handbook of Chaos Control" Wiley (1999)
(the most comprehensive coverage of the subject)

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Papers

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1. Arecchi, F.T. and Boccaletti, S. *Adaptive strategies for recognition, noise filtering, control, synchronization and targeting of chaos* (Chaos 7 no 4 621-634 1997)

2. Bielawski, S., Derozier, D. and Glorieux, P. *Experimental characterization of unstable orbits by controlling chaos* (Phys Rev A 47 no 4 1993)

3. Boccaletti, S., Grebogi C., Lai Y-C., Mancini, H. and Maza D. *The control of Chaos: Theory and Applicatons* (Phys Rep 329 103-197 2000)

4. Grebogi, C., Ott, E. and Yorke, JA *Unstable periodic orbits and the dimensions of multifractal chaotic attractors* (Phys. Rev. A 37, 1711 - 1724 1988)

5. Holyst, J.A., Hagel, T., Haag, G. and Weidlich, W. *How to control a chaotic economy* (Journal of Evolutionary Economics 6 1 31-42 March 1996)

6. Holyst, J.A. and Urbanowicz, K. *Chaos control in economic model by time-delayed feedback method* (Physica A 287 587-598 December 2000)

7. Hunt, E.R. *Stabilizing High-Period orbits in a Chaotic System: The Diode Resonator* (Phys Rev Lett 67 no 15 October 1991)

8. Lathrop, D.P. and Kostelich, E.J. *Characterization of an experimental strange attractor by periodic orbits*" (Phys Rev A 40 4028-4031 1989)

9. Ott, E., Grebogi C. and Yorke, JA *Controlling Chaos* (Phys Rev Lett 64 no 11 March 1990)

10. Packard, N.H., Crutchfield, J.P., Farmer, J.D. and Shaw, R.S. *Geometry from a Time Series* (Phys. Rev. Lett 45 no 9 712-716 1980)

11. Pyragas, K. *Continuous Control of chaos by self-controlling feedback* (Phys. Lett. A 170 421-8 1992)

12. Pyragas, K. *Control of Chaos via an Unstable Delayed Feedback Controller* (Phys. Rev. Lett 86 no 11 12th March 2001)

13. Shinbrot, T., Ott, E., Grebogi, C. and Yorke, J.A. *Using Chaos to Direct Trajectories to Targets* (Phys Rev Lett 65 no 26 3215-3218 1990)

14. Shinbrot, T, Ott, E., Grebogi, C. and Yorke, J.A. *Using chaos to direct orbits to targets in systems describable by a one-dimensional map* (Phys Rev A 45 no 6 4165-4168 1992)

15. So, P., Ott, E., Schiff, S.J., Kaplan, D.T., Sauer, T. and Grebogi, C. *Detecting unstable periodic orbits in chaotic experimental dat*" (Phys Rev Lett 76 4705-4708 1996)

16. Socolar, J.E .S., Sukow, D.W. and Gauthier D.J. *Stabilizing unstable period orbits in fast dynamical systems* (Phys Rev E 50 3245-3248 1994)